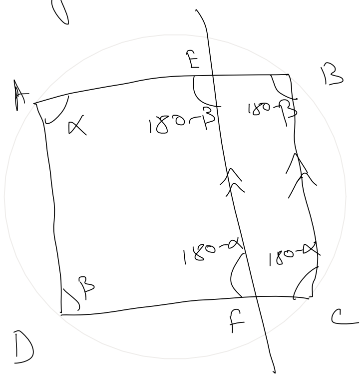
Q> Let ABCD be a cyclic quadrilateral. A line L parallel to BC cuts AB and CD at E and F respectively. Show that ADFE are concyclic.

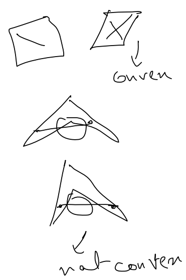
Ans:-

ADFE is concyclic $\Leftarrow$



Q> Let ABC be a triangle inscribed in a circle M. Show that $AC \perp CB$ iff $\overline{AB}$ is a diameter of M

Cyclic Quadrilaterals :-



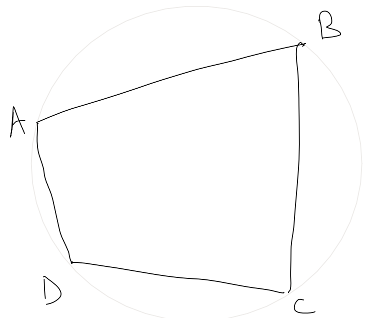
Theorem:- Let ABCD be a convex quadrilateral

Then the followings are equivalent:-

i> ABCD is cyclic

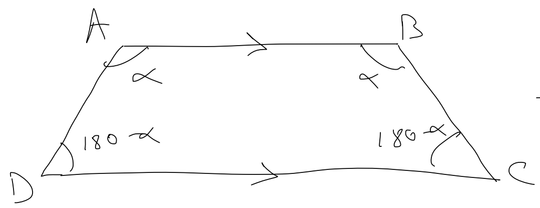
ii> $\angle ABC + \angle CDA = 180^\circ$

iii> $\angle ABD = \angle ACD$



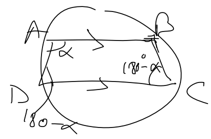
Q> Show that a trapezoid is cyclic iff it is isosceles.

Ans:-



$\Longrightarrow$ ABCD is cyclic

Now converse, let ABCD be a cyclic Trapezium



$\angle BAD = 180^\circ - \angle BCD$

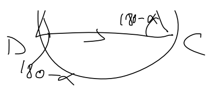
Also, $AB \parallel DC \Rightarrow \angle ADC = 180^\circ - \alpha$

$\Rightarrow \angle \ldots CDA = \alpha$

$\angle BAD = 180 - \angle BCD$

Also, $AB \parallel DC \Rightarrow \angle ADC = 180° - \alpha$

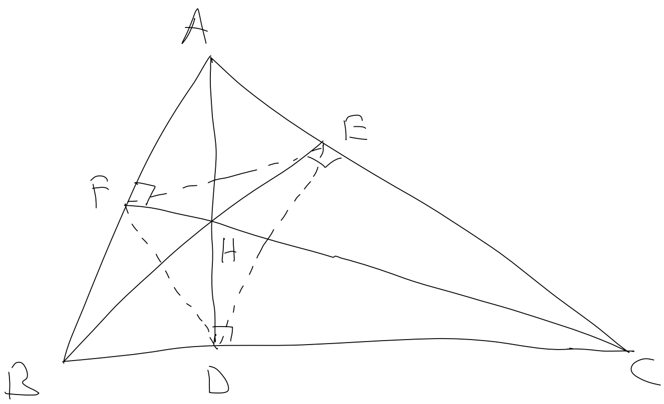
$\Rightarrow \angle CBA = \alpha$



$\Rightarrow ABCD$ is isosceles

Q) Quadrilateral $ABCD$ has $\angle ABC = \angle ADC = 90°$. Show that $ABCD$ is cyclic and that circumcircle of $ABCD$ has diameter $\overline{AC}$.

Orthic Triangle



DEF is the orthic triangle

H is the orthocentre

Homework

Lemma:- Suppose ΔDEF is the orthic triangle of acute ΔABC with orthocentre H. Then,

(i) Points A, E, F, H lie on a circle with diameter $\overline{AH}$

(ii) Points B, F, E, C lie on a circle with diameter $\overline{BC}$

(iii) H is the incentre of ΔDEF